OCR
Oxford Cambridge and RSA

Thursday 14 May 2015 – Morning

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Given that $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, where $\mathbf{M} = \begin{pmatrix} 4 & -3 \\ 8 & 21 \end{pmatrix}$, find x and y. **[6]**

2 Find the roots of the quadratic equation $z^2 - 4z + 13 = 0$.

Find the modulus and argument of each root. **[5]**

3 The equation $2x^3 + px^2 + qx + r = 0$ has a root at $x = 4$. The sum of the roots is 6 and the product of the roots is -10. Find p, q and r. **[6]**

4 Indicate, on a single Argand diagram

(i) the set of points for which $\arg(z - (-1 - \mathrm{j})) = \frac{\pi}{4}$, **[2]**

(ii) the set of points for which $|z - (1 + 2\mathrm{j})| = 2$, **[2]**

(iii) the set of points for which $|z - (1 + 2\mathrm{j})| \geqslant 2$ and $0 \leqslant \arg(z - (-1 - \mathrm{j})) \leqslant \frac{\pi}{4}$. **[2]**

5 **(i)** Show that $\sum_{r=1}^{n} (2r - 1) = n^2$. **[3]**

(ii) Show that $\dfrac{\sum_{r=1}^{n} (2r - 1)}{\sum_{r=n+1}^{2n} (2r - 1)} = k$, where k is a constant to be determined. **[4]**

6 A sequence is defined by $u_1 = 3$ and $u_{n+1} = 3u_n - 5$. Prove by induction that $u_n = \frac{3^{n-1} + 5}{2}$. **[6]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{(3x+2)(x-3)}{(x-2)(x+1)}$.

(i) Write down the equations of the three asymptotes and the coordinates of the points where the curve crosses the axes. **[4]**

(ii) Sketch the curve, justifying how it approaches the horizontal asymptote. **[5]**

(iii) Find the set of values of x for which $y \geqslant 3$. **[3]**

8 The complex number 5 + 4j is denoted by α.

(i) Find α^2 and α^3, showing your working. **[3]**

(ii) The real numbers q and r are such that $\alpha^3 + q\alpha^2 + 11\alpha + r = 0$. Find q and r. **[4]**

Let $\mathrm{f}(z) = z^3 + qz^2 + 11z + r$, where q and r are as in part **(ii)**.

(iii) Solve the equation $\mathrm{f}(z) = 0$. **[3]**

(iv) Solve the equation $z^4 + qz^3 + 11z^2 + rz = z^3 + qz^2 + 11z + r$. **[2]**

9 The triangle ABC has vertices at $\mathrm{A}(0,0)$, $\mathrm{B}(0,2)$ and $\mathrm{C}(4,1)$. The matrix $\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$ represents a transformation T.

(i) The transformation T maps triangle ABC onto triangle $\mathrm{A'B'C'}$. Find the coordinates of $\mathrm{A'}$, $\mathrm{B'}$ and $\mathrm{C'}$. **[3]**

Triangle $\mathrm{A'B'C'}$ is now mapped onto triangle $\mathrm{A''B''C''}$ using the matrix $\mathbf{M} = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$.

(ii) Describe fully the transformation represented by **M**. **[3]**

(iii) Triangle $\mathrm{A''B''C''}$ is now mapped back onto ABC by a single transformation. Find the matrix representing this transformation. **[3]**

(iv) Calculate the area of $\mathrm{A''B''C''}$. **[3]**

END OF QUESTION PAPER